

Book Review

Anatoly G. Kusraev and Seman S. Kutateladze, *Subdifferentials: Theory and Application*, Mathematics and its Applications Vol. 323, Kluwer Academic Publishers, Dordrecht – Boston – London, 1995 (408 pp. Hb ISBN 0-7923-3389-6).

This monograph deals with a new area in functional analysis, the subdifferential calculus and its applications. It is well-known that this calculus is based on a generalization of the notion of classical differentiation for nonsmooth functionals. This generalization results in a natural way from the need for rational treatment of optimization problems. The book contains an in depth presentation of the most important results of the subdifferential calculus both in a standard and in a non-standard framework. New tools and methods of convex nonsmooth analysis are discussed such as, Kantorovich spaces, Boolean-valued and infinitesimal versions of nonstandard analysis etc.

The first Chapter deals with convex correspondences and operators, as well as with the fans of Ioffe, and systems of convex objects. In the second Chapter the geometrical nature of the subdifferentials is investigated. This Chapter deals with the method of canonical operators, the extreme and the intrinsic structure of subdifferentials, with the notion of caps and faces and their properties etc.

In the third Chapter the relation between convexity and openness is studied. Sections concerning the calculus of polars, the dual characterization of openness and the openness and completeness are included among others. The fourth Chapter is the main chapter of the book, because therein the apparatus of Subdifferential Calculus is presented. After the Young-Fenchel transform the authors give formulas for subdifferentiation and they deal with the semicontinuity and the Maharam operators. The chapter closes with a section on the disintegration and a section on infinitesimal subdifferentials. The fifth Chapter considers convex extremal problems, with emphasis on vector programs, the Lagrange principle, the conditions for optimality, approximate optimality and infinitesimal optimality. Moreover the notion of generalized solutions is appropriately discussed. The last Chapter has the title “Convex Local Approximations”. It deals with the Kuratowski and Rockafellar limits, the approximation by means of a set of infinitesimals and closes with the subdifferentials of nonsmooth operators.

The References are complete as well as the author index and the subject index. The present volume fills the gap between the existing today theoretical core of modern functional analysis and the more applied subjects, such as optimization, optimal control, economics etc.

The book is written with clarity and great originality and contains new material, which is of interest to pure mathematicians, to applied mathematicians, as well as to all scientists working on Optimization. It is indeed a very interesting book written by the appropriate scientists in the right time.

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